

Recurrence and the Seam

Recurrence and oscillator methods, applied to the seam

From two drafts by Dr. Charles Jorgensen (NASA Ames, ret.): how a single measured variable, the heart included, reconstructs a hidden state; why three oscillators lock at 120° ; and where the heart's role stops

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Abstract

This note connects two unpublished drafts by Dr. Charles Jorgensen (NASA Ames, ret.), one on recurrence analysis of pilot heart-rate records and one on a computing architecture built from coupled oscillators, to the scaled-metric reading of the three-orb paper. It finds four points of contact, in descending order of strength. Recurrence quantification analysis is the right instrument for the jitter test the supplement proposes in S3.3: it separates a control-loop residual, a non-extremal residence and a keyed animation by published, validated measures, and worked examples are given, with surrogate tests, a false-nearest-neighbor estimate of the hidden dimension, the lock threshold and breaking route of three coupled oscillators, cross-recurrence for deciding which of several stored reference records a new sample matches, and the dwell scaling of a heart-rate record. Takens embedding supplies the mathematics the reference instrument (design study) lacks: from a single measured scalar, the phase differential, the hidden dimension of the medium's dynamics can be estimated, which gives the framework's dimension number an empirical counterpart. The 120° triad is the splay state of three repulsively coupled phase oscillators, which yields a dynamical model of the lock, a threshold for it, and a relaxation signature the footage can be checked against. And the heart enters on the measurement side: a heart-rate record, embedded and quantified, gives a within-person reference against which later records can be compared, and under the framework a dwell at clock-rate ratio r should return that person's own recurrence periods scaled by r . The note states plainly what the papers do not supply: any coupling from a biological oscillator to the scaling field. The heart measures; it does not open the seam. A final section, prompted by a suggestion from Dr. Norbert Schwarzer, adds a test for a dominating scale that separates a seam from a system nearing a transition, and an early-warning procedure for a single record.

Status. Jorgensen's two drafts [R1, R2] are unpublished and are cited as unpublished drafts, with his permission. Jorgensen reviewed and approved this note and the interactive pieces that accompany it. Dr. Norbert Schwarzer has not reviewed them, and any error in applying the methods to a seam is mine. Everything said here about a seam is an exercise and a set of predictions, not a finding. The main paper is cited as *Geometry of a Three-Orb Metastructure*, Revision 10.2, the reissue that incorporates N. J. Bocker's review; its section and equation numbers, used throughout, are unchanged from Revision 10.1.

R1. What the two drafts establish

Recurrence analysis of pilot schedules. In a joint NASA and easyJet study, commercial pilots wore a chest harness that logged heart rate, chest temperature and three-axis motion over three blocks spanning fifteen days. The heart-rate series was embedded by Takens' theorem into a reconstructed state space, with the embedding dimension chosen by global false nearest neighbors and the delay by mutual information, and the resulting recurrence plots were quantified by the standard measures: recurrence rate, determinism, laminarity, trapping time, maximal line, entropy and trend. The draft's preliminary finding is that the recurrence histogram has distinct periods in flight (about 30 min, 115 min and 17 h for one pilot) and different, longer ones out of flight, for two of four pilots. The method is the point: a single observable suffices to reconstruct the geometry of a hidden process and to count its dimensions.

A bio-oscillator computing architecture. The second draft proposes computation from canonical nerve-cell models: bursters that respond to temporal patterns, resonators that amplify them briefly, and self-excited oscillators that phase-lock through pulse coupling into persistent clusters the paper calls modes. Modes are treated as potential fields on a small-world network, with the statistical mechanics of such networks doing the bookkeeping, and the paper cites MacLennan's field computers as the formal setting. Its reported status is working demonstrations of frequency locking, selective communication and phase locking, and attractor-based logic circuits.

R2. Recurrence quantification as the jitter test

The supplement's sharpest prediction about the formation (S3.3) is that a triad held by a control loop drives its residual to sensor noise, while a triad at a non-extremal residence keeps a jitter floor with structure; the main paper (§7.2) added that a keyed animation shows neither but a smooth interpolation. I proposed a power spectrum. Recurrence quantification is better suited, because it was built to tell short deterministic series from stochastic ones [R3] and because its measures map directly onto the three readings. Figure R1 shows the three cases on synthetic spacing residuals of 600 frames at 24 fps, embedded at $m = 3$ and $\tau = 4$ with a radius of 0.6σ .

Figure R1. Recurrence plots of three readings of the triad's spacing residual (Takens embedding $m = 3$, $\tau = 4$ frames, radius 60 % of σ , main diagonal excluded)

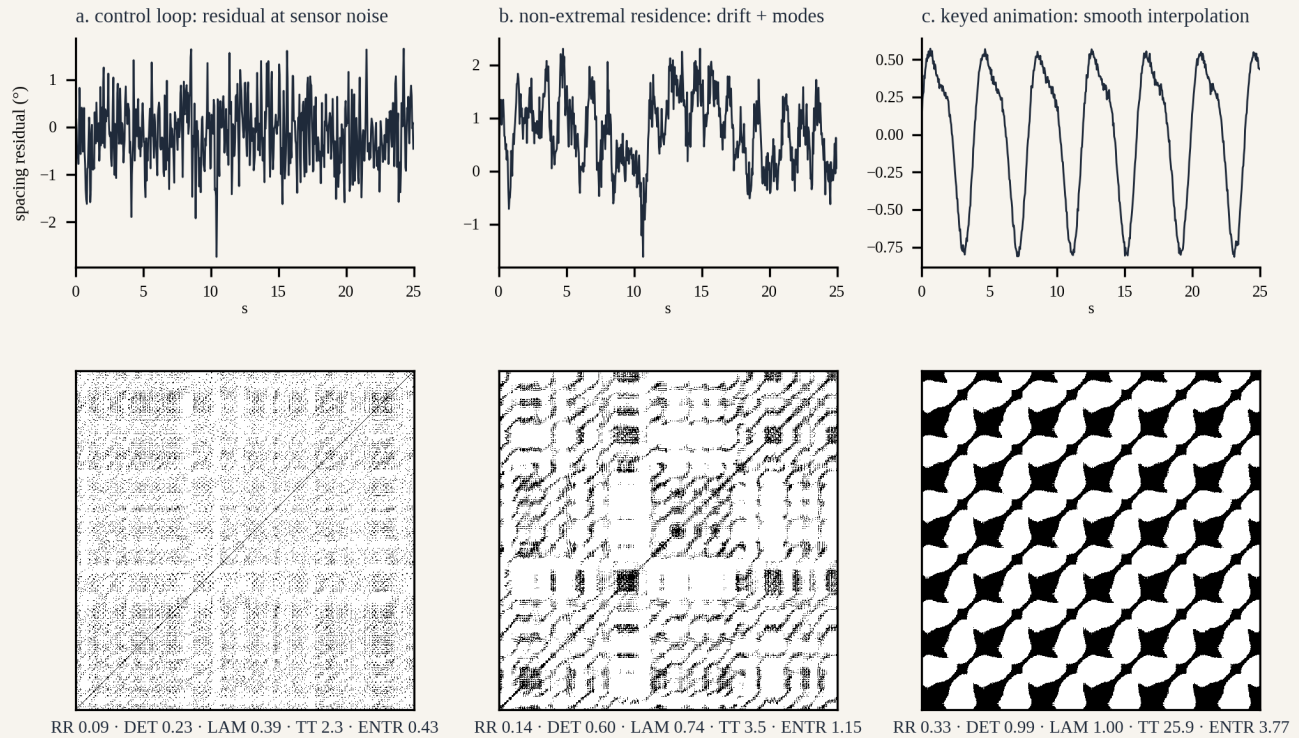


Figure R1. Recurrence plots of three readings of the triad's spacing residual. Left, a control loop: the residual is near-white, the plot is homogeneous, determinism low. Center, a non-extremal residence: drift about an offset carrying the two spring modes of Equation 8; the plot shows laminar blocks and interrupted diagonals, determinism and laminarity intermediate, entropy raised. Right, a keyed animation: a smooth periodic interpolation; the plot is a lattice, determinism and laminarity near one, the longest diagonal spans the record.

Measure	Control loop	Non-extremal residence	Keyed animation	Reads as
Recurrence rate	0.09	0.14	0.33	fraction of state pairs that recur
Determinism	0.23	0.60	0.99	fraction of recurrences on diagonal lines
Laminarity	0.39	0.74	1.00	fraction on vertical lines (dwelling)
Trapping time	2.3	3.5	25.9	mean dwell, frames
Longest diagonal	8	23	591	frames; $\approx$ record length means periodic
Entropy of line lengths	0.43	1.15	3.77	structure in the recurrences

Table R1. RQA measures for the three synthetic readings of Figure R1. The numbers are illustrative of the separation, not thresholds; real thresholds come from surrogate tests on the tracked frames.

Three things follow. First, the test is cheap: it runs on the same tracked spacing and radius series §16.1 already asks for, and the software is public (Webber's RQA toolbox, Marwan's CRP toolbox). Second, it discriminates the animation reading as well as the physics readings, which the spectrum did not do cleanly: an interpolated path has a recurrence lattice and a longest diagonal that spans the record. Third, it gives Schwarzer's fulcrum parameter H , which the framework leaves free, an empirical handle: at a non-extremal residence the recurrence plot carries trend and laminarity that a residence at the extremum lacks, so the entropy and trend of the plot are a candidate measurement of H . The addition to §16.1 and S3.3 is one sentence each.

R2.1 Procedure for the footage

1. Track the three sources frame by frame on the best copies (§16.1) and form two series per frame: the departure of each pair's angular spacing from 120° , and the departure of the radius from its running mean. Detrend only the radius, never the spacing.
2. Choose the delay by the first minimum of the mutual information of each series, and the embedding dimension by global false nearest neighbors (R3). For a 24 fps record of a few hundred frames, expect τ of a few frames and m between 3 and 5.
3. Set the recurrence radius at a fixed fraction of the series' standard deviation (0.5 to 1.0σ) and report the measures at two radii, so that no conclusion rests on the choice.
4. Compute RR, DET, LAM, TT, $L_{\max}$ and ENTR with a published toolbox, and compute the same measures on at least 100 surrogates of each series: shuffled (destroys all structure), phase-randomized (keeps the spectrum, destroys nonlinear structure), and a smooth spline fitted through the keyframes (the animation hypothesis).
5. Read the result against Table R1. A series whose DET and LAM sit inside the shuffled-surrogate band is the control-loop reading; one that exceeds the phase-randomized band is the residence reading; one whose $L_{\max}$ approaches the record length and whose measures match the spline surrogate is the animation reading.

The surrogate step is what makes the numbers in Table R1 into a test. On their own the measures say only that the three synthetic series differ; against surrogates they say which hypothesis a real series is consistent with, at a stated confidence.

R3. Takens embedding and the reference instrument

The reference instrument of the design study reads one scalar, the phase differential $\sqrt{(F/F_b)}$, and the design study treats the state of the medium at the instrument as hidden, to be inferred from that scalar. Takens' theorem is the mathematics that makes such a claim respectable: for almost any smooth observation function, the delay-coordinate map of a single measured variable is an embedding of the underlying attractor, so its dimension and its recurrence structure can be estimated from the one channel. The procedure is Jorgensen's: estimate the embedding dimension of the phase-differential series by false nearest neighbors, the delay by mutual information, and quantify. The number that comes out, the embedding dimension M , is a measured count of the hidden degrees of freedom of the medium at the instrument. Schwarzer's n is a count of parameter dimensions in the equations. Whether M and n have anything to do with each other is an empirical question, and it is the kind of question the lab exists to pose: the framework predicts that M changes across a seam, since the dominant term changes; a fixed-stage physics predicts it does not.

Figure R3. Global false-nearest-neighbor fraction against embedding dimension for the three readings ($\tau = 4$, tolerance 15)

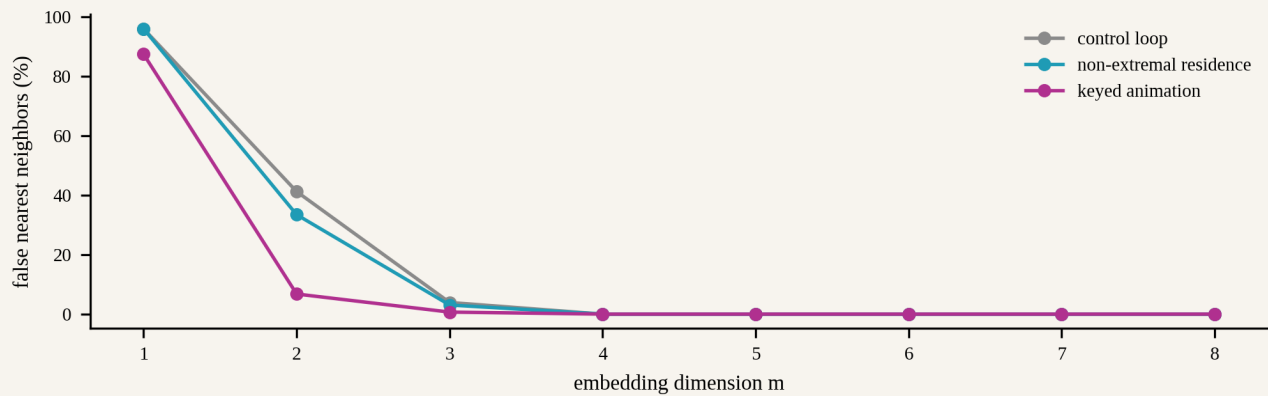


Figure R3. False-nearest-neighbor fraction against embedding dimension for the three readings of Figure R1. The animation unfolds at $m = 3$; the two physical readings need $m = 4$. The dimension is a property of the generating process that survives the choice of radius and is the first number a tracked series should yield.

On the synthetic series the fraction of false neighbors falls to zero at $m = 3$ for the keyed animation and at $m = 4$ for the two physical readings (7 % against 41 % and 33 % at $m = 2$). A smooth interpolation between keyframes lives on a low-dimensional manifold by construction; a residual with modes, drift and noise needs one more dimension to unfold. That difference is a second animation test independent of the recurrence measures, and it costs nothing beyond the embedding already done.

R4. The triad as a splay state

Jorgensen's architecture is about oscillators that lock through pulse coupling. Three identical phase oscillators coupled all-to-all by a sine of their phase differences have two locked states, in-phase and splay, and the splay state puts the three phases equally spaced on the circle. Three equally spaced phases are the triad of Equation 1. Linearizing about the splay state, the perturbations obey

$$\dot{\theta}_i = \omega + \frac{K}{3} \sum_{j \neq i} \sin(\theta_j - \theta_i), \quad \theta_i^* = \theta_0 + \frac{2\pi i}{3}, \quad \dot{\delta}_i = \frac{K}{3} \cos\left(\frac{2\pi}{3}\right) \sum_{j \neq i} (\delta_j - \delta_i) = \frac{K}{2} \delta_i \quad \left(\sum \delta = 0\right)$$

Equation R1. Three sine-coupled phase oscillators, their splay state, and the growth rate of zero-mean perturbations about it.

so the splay state is stable precisely when the coupling is repulsive, $K < 0$, with relaxation rate $|K|/2$. That is a result worth pausing on, because it reconciles two pictures the main paper left in tension. Section 9 noted that the commentary's “outward magnetic pressure holding the triangle apart” gives repulsion, and §7.1 that a rotating triad needs net attraction to the center. In the oscillator model those are different quantities: a repulsive *phase* coupling holds the three at 120° in phase, while the *radial* binding is supplied separately, by the gradient energy of the scaling field in S3.2. The 120° lock is then not a coincidence of a control law but the generic locked state of three mutually repelling oscillators, and it has a signature: after a disturbance the separations return to 120° exponentially, with one time constant $2/|K|$, rather than ringing as the spring modes of Equation 8 would. Figure R2 shows the recovery for $K = -0.8 \text{ s}^{-1}$. The footage test is the relaxation after any visible jolt: exponential return names an oscillator lock, ringing names a spring binding, no return names an animation.

Figure R2. Three repulsively coupled phase oscillators ($K = -0.8 \text{ s}^{-1}$) hold the splay state at 120° and recover from a kick

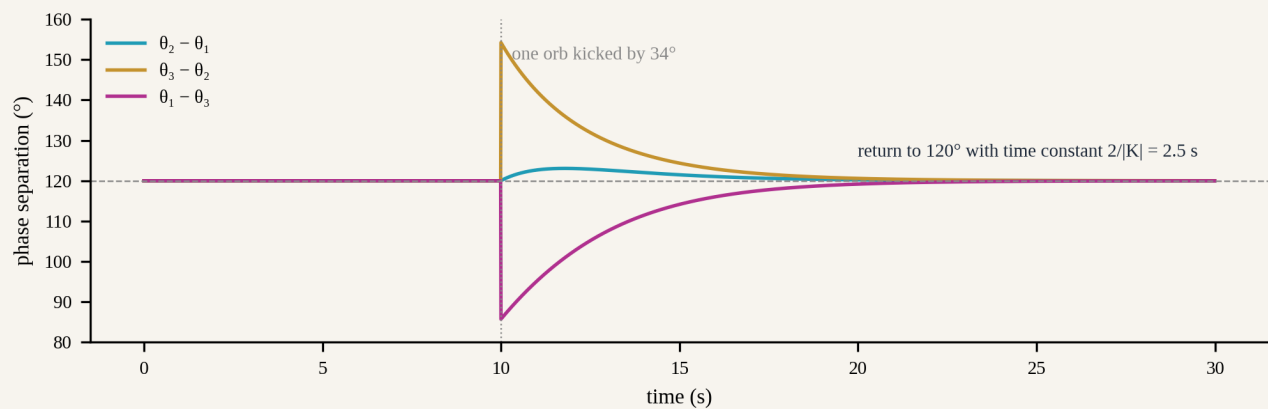


Figure R2. Three repulsively coupled phase oscillators hold the splay state. One is kicked by 34° at $t = 10 \text{ s}$; the separations return to 120° with time constant $2/|K| = 2.5 \text{ s}$ and no overshoot. A spring-bound triad would ring at the frequencies of Equation 8.

Jorgensen's modes add one more thing: the lock forms only above a coupling threshold set by the oscillators' frequency spread, and it breaks by a specific route, one oscillator dropping out of the cluster and then the pair. The collapse of §5 could be read against that route frame by frame.

Figure R4. Threshold and breaking route of the three-oscillator lock with a frequency spread

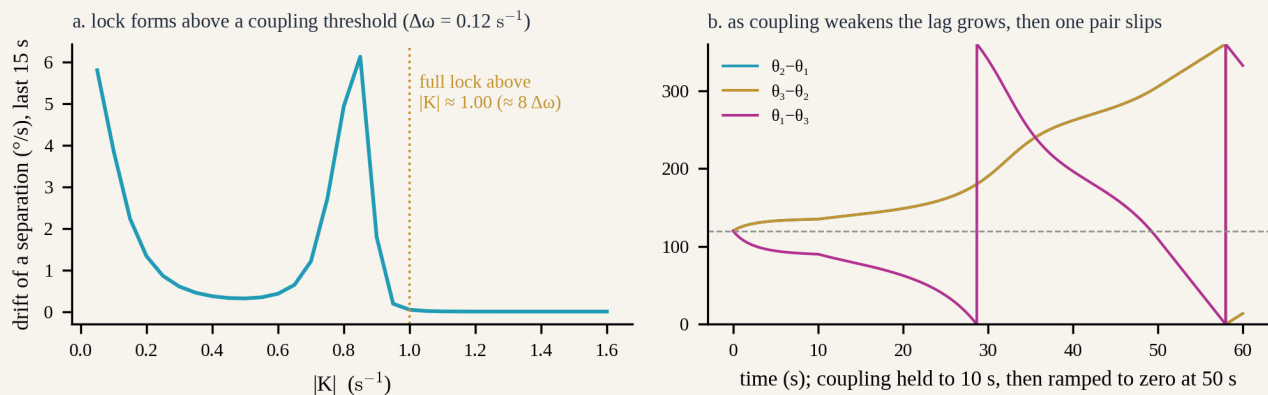


Figure R4. Left: with a spread of natural frequencies ($\Delta\omega = 0.12 \text{ s}^{-1}$ between neighbors) the three oscillators lock fully only above a coupling threshold, with a partially locked band below it in which two hold and one drifts. Right: as the coupling is ramped toward zero the locked separations lean away from 120° and then one pair slips, which is the breaking route.

With a spread of natural frequencies the lock has a threshold: for the spread in Figure R4 full lock needs $|K|$ above about 1.0 s^{-1} , and below it there is a band in which two oscillators hold and the third drifts. The route to breaking is the reverse of the route to locking: as coupling weakens the locked separations lean away from 120° in proportion to $\Delta\omega/|K|$, and when the lean reaches 90° one pair slips. The main paper's §5 describes the collapse as a tightening of the triangle; the oscillator model says the phase lock should fail first by a visible lean of the separations away from 120° , then by one source slipping a full turn relative to the other two, and that a formation which collapses radially while holding exactly 120° to the end is not an oscillator lock at all. That is a frame-by-frame prediction the footage can meet or fail.

Jorgensen's element	Function in his architecture	Counterpart in the seam reading	Where it is tested
Burster	Responds to an extended temporal and phase pattern from outside	The trigger: whatever sets the scaling field's boundary condition; not modeled	—
Resonator	Amplifies a small specific pattern for a limited time	The transient that forms a coherent domain; the fireball of §16.2 is the laboratory case	Chamber
Oscillator	Self-excited; locks through pulse coupling; persists	A coherent domain of the medium (P5); three of them are the triad	Footage: splay state, relaxation
Mode	A locked cluster; a potential field at one time scale	The locked triad and its binding potential (S3.2); the seam when the mode spans the boundary	Footage: lock threshold, breaking route
Micro-behavior	Threshold crossing that fires a controller	The flash: the mode's termination	Footage: flash frames
Small-world statistics	Bookkeeping of many weakly coupled cells	Not needed for three domains; would govern a formation of many	—

Table R2. Jorgensen's architecture mapped onto the seam reading. The mapping is structural, not a claim that the medium is a nerve net.

R5. The heart: where it enters and where it stops

The reference instrument of the design study reads the medium against a stored reference. A person can be read the same way: a heart-rate record compared, not with anyone else's, but with that person's own earlier records taken under known conditions. Jorgensen's two drafts together say how such a within-person reference could be built, and where its role stops.

How it could. Recurrence analysis gives a within-person reference a quantitative form: embed the person's heart-rate series, quantify, and the six measures of Table R1, taken over several quiet sessions, are the reference band. The harder case is a sample that must be assigned to one of several stored records when each of them is settled and well ordered, so that the single-record measures cannot tell them apart. That is a cross-recurrence problem, and Marwan and Kurths' cross-recurrence plots [R4] are the published tool for it: cross-quantify the new sample against each stored record, and the record it recurs with is the one it matches.

His own precedent. Jorgensen has patented this principle for a neighboring purpose, the verification of identity. In U.S. Patent 8,924,736, with S. V. Dusan, a person is enrolled by finding the heartbeats in a recording that most resemble one another and combining them into a reference; a later recording is then accepted or declined by its distance from that reference [R12]. In U.S. Patent 8,489,181, with J. F. Schipper, S. V. Dusan and E. Belousof, sixteen measurements of each heartbeat (peak amplitudes, intervals between waves, and the angles and lengths of the depolarization and repolarization vectors) are taken on up to twelve channels and summarized over many beats by their normalized mean, spread, skewness and kurtosis, as many as 192 statistics compared with the person's

stored statistics within acceptance ranges [R13]. Both compare a person with his or her own record, not with a population. The within-person band proposed here applies the same logic to the recurrence measures of the rhythm rather than to the shape of single beats. The two are complementary, and a reference band could carry both.

Figure R5. Cross-recurrence plots of a two-minute heart-rate sample against two stored reference records (synthetic)

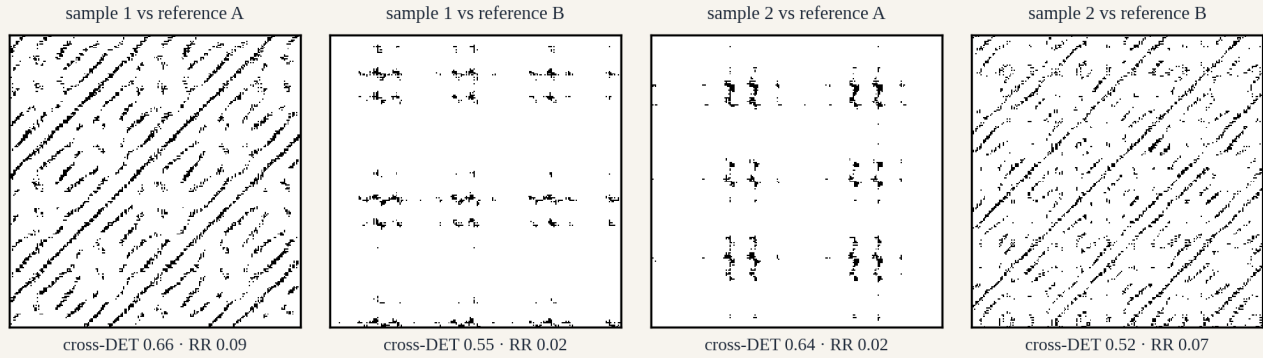


Figure R5. Cross-recurrence of a two-minute heart-rate sample against two stored reference records, A and B (synthetic). A sample drawn from the same process as A recurs with A and not with B; a sample drawn from B's process does the reverse. The cross-recurrence rate, not the determinism, carries the decision.

On the synthetic records the cross-recurrence rate separates the cases by a factor of four or more (0.09 against 0.02 for a sample from A's process; 0.02 against 0.07 for one from B's), while the cross-determinism barely moves. That is the expected behavior: determinism measures how deterministic the two records are, which both are, and recurrence rate measures whether they visit the same states, which is the question. The decision rule is then: compute the cross-recurrence rate of the sample against every stored record; the record it recurs with is the one it matches, whatever the single-record measures look like.

A prediction about time. A seam moves no date, but a dwell at clock-rate ratio r stretches every internal clock by r . Jorgensen found recurrence periods of about 30 minutes, 115 minutes and 17 hours in a pilot's heart-rate record [R1]. If the framework is right, those periods should return scaled by r on the same person's record after a dwell, so the body would be a clock that measures the accumulated offset $(r - 1) \cdot \tau$ independently of the reference instrument. It needs only the chest harness the pilots in [R1] already wore.

Figure R6. Recurrence histogram of a heart-rate record before and after a dwell at clock-rate ratio $r = 1.5$ (synthetic)

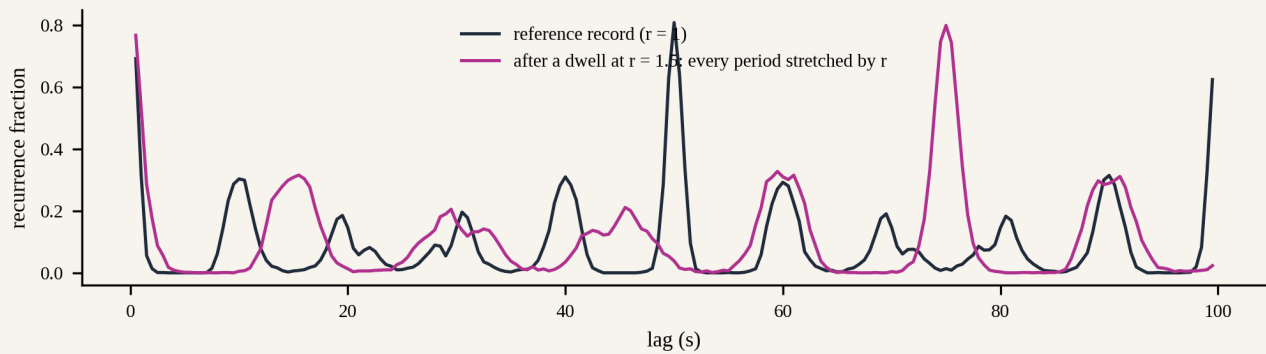


Figure R6. Recurrence histogram of a heart-rate record before and after a dwell at clock-rate ratio $r = 1.5$ (synthetic). Every recurrence period returns stretched by r ; the dominant peak moves from 50 s to 75 s of lag. Under the framework, the body's own rhythms would carry the accumulated offset.

On a synthetic record the dominant recurrence peak moves from lag 100 to lag 150 samples for $r = 1.5$, which is the ratio exactly. Two cautions apply to a real one. Heart-rate periods also move with sleep, stimulants and workload, which is the point of Jorgensen's study, so the comparison must be made against the person's own reference histogram under matched conditions, and the prediction is a uniform scaling of all periods together, which ordinary physiology does not produce. A uniform stretch of the whole histogram by one factor is the signature; any one period moving on its own is physiology. A record that returned unscaled after a dwell at $r \neq 1$ would put the two clocks in disagreement, and the exercise would have failed in a way that can be seen.

Where it stops. Nothing in either draft supplies a coupling from a biological oscillator to the scaling field. The reference instrument of the design study is not a biomagnetic sensor, and nothing in this note proposes that the body carries information about the scaling field. The supplement's S7 reached the same line from the physics side: the seventy-eight orders of magnitude between the energy of Equation 14 and any biological signal are untouched by anything here. The heart's role is measurement. It can be embedded and quantified with validated tools, and under the framework it carries a prediction about dwell. It does not open the seam, and the note would be weaker, not stronger, for saying otherwise.

R6. Dominating scales and early detection of change

Jorgensen opened his pilot draft with one sentence: “An important problem in the study of complex systems is early detection of change” [R1]. Dr. Norbert Schwarzer, reading it, suggested where to look: at the existence of dominating scales [R8], with a pointer to the renormalization-group volume of Domb and Green's series on critical phenomena [R9]. The lesson of that theory fits in three lines. Away from a transition a system has a dominating scale, a characteristic length or time over which it forgets a disturbance. Approaching a transition the scale grows, because recovery slows. At the transition no single scale dominates. Ecology and climate science use the middle line as an early warning of tipping points [R10, R11]. The suggestion does two things for this note: it gives the seam test a third outcome, and it gives the heart-rate method a use that does not need a seam at all.

A third verdict for the seam test. The console's seam invariance test (instrument 01·F) compares the same process recorded on each side of a seam. Scale-carrying measures (embedding delay, recurrence return time, trapping time, dominant period) must stretch by the clock-rate ratio r ; shape measures (embedding dimension, D_2 , DET, LAM) must not move. To these I add one measure of the dominating scale. For a record with a dominant period T_{dom} , the autocorrelation oscillates inside an envelope that decays over a time T_{env} ; the sharpness $Q = \pi T_{\text{env}}/T_{\text{dom}}$ is a ratio of two times, so a seam cannot change it. A process whose recovery is slowing keeps its period and raises Q . A process whose dynamics changed alters its shape or brings in a different period. The test therefore reads three cases where it read two: a pure seam, changed dynamics, and a system nearing a transition (Figure R7, a and b).

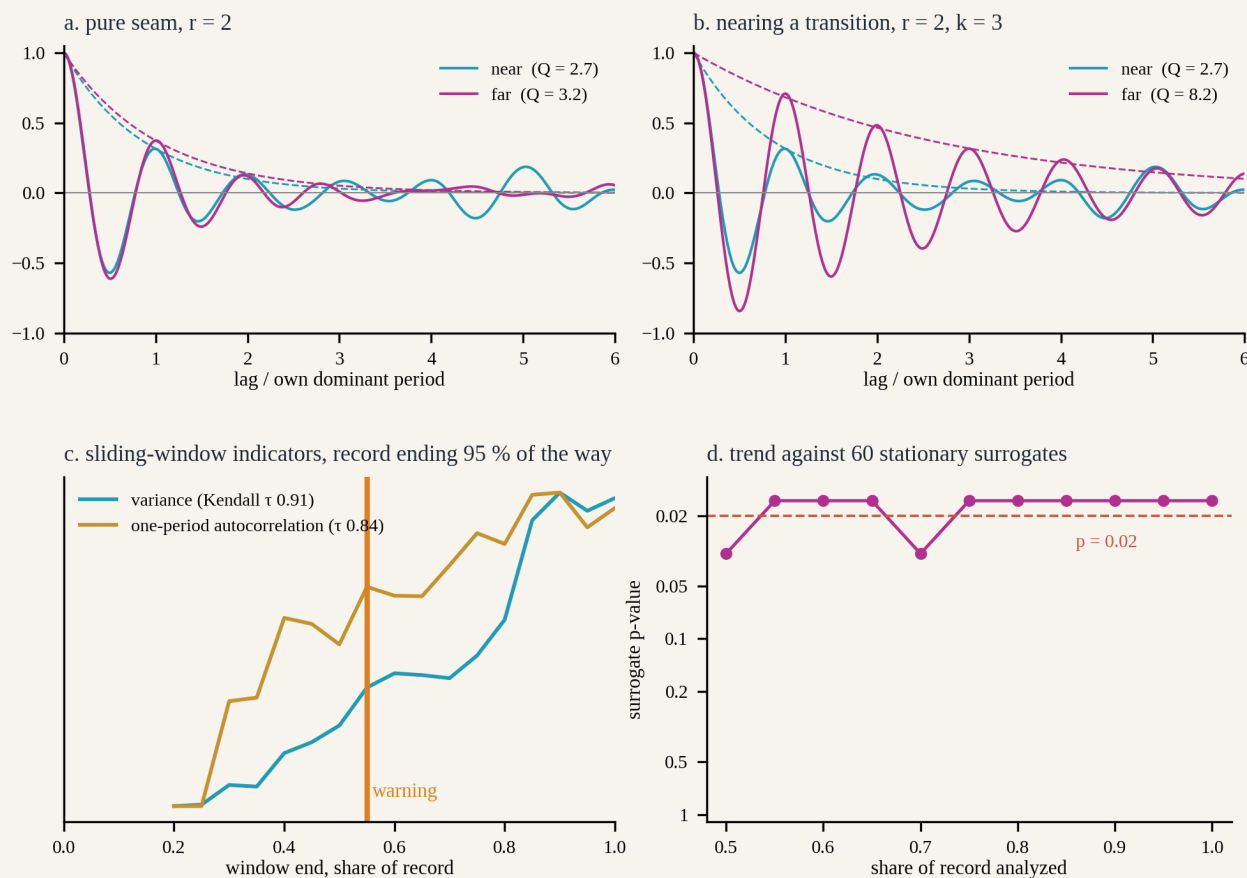
Figure R7. Dominating scales: a seam leaves the sharpness Q alone; slowing recovery raises it and is caught early

Figure R7. (a) A noise-driven rhythm recorded across a pure seam ($r = 2$): with lag measured in each side's own dominant period, the two autocorrelations and their envelopes coincide, and Q holds. (b) The same rhythm with its damping divided by $k = 3$ on the far side: the period still stretches by r , but the envelope decays over more cycles and Q rises about threefold. (c) A single record whose damping falls toward zero, ending 95 % of the way to the transition: variance and one-period autocorrelation in sliding windows. (d) The trend against 60 phase-randomized surrogates as more of the record is analyzed; the warning is raised at the first of two consecutive checkpoints below $p = 0.02$. All synthetic.

On synthetic records the three-way test returned 267 correct verdicts in 270 trials. The trials covered four processes (Lorenz, Rössler, a heart-like quasi-periodic rhythm and a noise-driven rhythm), six random seeds, and r from 1 to 4, with the damping divided by $k = 3$ in the transition cases. Two pure seams read as changed, and one raised an early warning. A firm transition verdict requires Q to rise by $\times 1.8$, which comes once the recovery time has roughly doubled: at $k = 2$, 26 of 30 runs were flagged, 16 of them firmly. An early-warning tier (Q up $\times 1.45$ to $\times 1.8$) catches the change sooner, at the cost of about one false alarm in ten pure-seam runs of the noise-driven rhythm. The Lorenz flow has no dominating scale, and the test reports the Q check as not applicable, which is itself the information Schwarzer's hint asks for.

Early warning within one record. The same idea works without a seam. In sliding windows over a single record, compute the variance and the autocorrelation at one dominant period; under slowing recovery both rise. A rising trend proves nothing by itself, because a steady rhythm also wanders, so the trend (Kendall's τ , averaged over the two measures) is compared with the same trend on 60 phase-randomized surrogates that share the record's spectrum but are stationary [R11]. The record is tested at eleven checkpoints, from half its length to all of it, and a warning is raised at the first checkpoint where the trend beats the surrogates ($p < 0.02$) twice in a row. Table R3 gives the result on 8,000-sample records of a rhythm whose damping falls linearly toward zero.

Record ends at	Warnings raised (of 20)	Median warning point	Reads as
steady rhythm (control)	2	—	false-alarm rate about one in ten
50 % of the way to the transition	8	43 % of the way	weak drift, seldom caught
70 % of the way	11	60 % of the way	
85 % of the way	12	68 % of the way	
95 % of the way	14	81 % of the way	caught in most records, well before the transition

Table R3. Early warning on synthetic records (20 seeds each). The warning point is the share of the way from the start of the record to the transition at which the warning was first raised. An early warning is a probability, not a certainty.

Making change visible. Jorgensen's change-detection program had a display side as well as a measurement side; his pilot draft names human data visualization as its primary goal [R1]. With S. Sharma he presented two papers in 2009 on visual coding of analog signals, and of monotonic signals and closed functions, for advanced displays [R14, R15], and NASA published his method for the visualization of analog signals as a Tech Brief in 2014, describing its use to identify changes or faults in a system [R16]. A related patent shows how strongly each of many documents carries a chosen dimension as small figures coded by color, texture or size [R17]. The Unfolding Bench now carries a display in that direction: each sliding window is drawn as its own closed loop, the signal against itself a quarter of a period later, on a disc shaded by the window's rank on the two early-warning measures, so that slowing recovery shows as loops that grow and close into cleaner rings. It is built from the published descriptions only and does not reproduce the method of [R14, R15].

Where the heart comes in. The recurrence periods Jorgensen found in a pilot's heart-rate record, about 30 minutes, 115 minutes and 17 hours [R1], are dominating scales. The exercise predicts that a change of state in such a record, a slide into fatigue for instance, would be preceded by those peaks sharpening, or by a joint rise in variance and one-period autocorrelation in sliding windows. Whether it is preceded so is an empirical question that records like those of [R1] could answer. Marwan and colleagues reported a related result for the heart: measures built on laminar phases, the vertical structures of the recurrence plot, picked up changes in heart-rate variability before the onset of life-threatening arrhythmias [R4].

Where SLE comes in. The Unfolding Bench reads the Schramm–Loewner parameter κ from a curve by unzipping it and plotting $\hat{\kappa}$ against scale. The plateau in that plot is a direct check that no dominating scale is present over that range, which is the only regime in which SLE applies. Below the plateau $\hat{\kappa}$ falls away, and the scale at which it first reaches half its plateau value is a crossover length ℓ^* : a pixel size, or a correlation length in the data, a dominating scale. Stretching a curve by $\times 2$, as a seam with $r = 2$ would, leaves $\hat{\kappa}$ unchanged and doubles ℓ^* (on the synthetic chest-field map, $\hat{\kappa} 4.17 \rightarrow 4.17$ and $\ell^* 1.49 \rightarrow 2.98$). The result is exact by construction, because the zipper is scale-covariant; what it adds is the reading. κ is a shape measure, and the crossover is a ruler. That answers the earlier conclusion that κ cannot measure a seam: κ cannot, but the crossover can.

An analogy, not a derivation. In renormalization-group theory the dimension of space enters the results explicitly, and above an upper critical dimension fluctuations stop mattering and the classical mean-field description becomes exact [R9]. Schwarzer's dimension number n also enters his equations as a coefficient, and the console's dimension dial shows the classical stationary principle returning at large n . The resemblance is offered for Schwarzer's judgment. Nothing here derives one from the other.

R7. Additions to the release

- §16.1 and S3.3: replace “a spectrum of the residual jitter” with “a recurrence quantification of the residual jitter (determinism, laminarity, trapping time, entropy), with surrogate tests,” and cite [R1]–[R3].
- S3.2: add the splay-state result of Equation R1 as the dynamical model of the 120° lock, with the relaxation signature as a fifth distinguishing observation.
- Console (done in this release): instrument 01·D, a recurrence bench that embeds a pasted or synthetic series, reports the false-nearest-neighbor curve and the six measures live, with the three readings of Figure R1 as presets; and instrument 01·E, a splay-state lock with coupling and frequency-spread dials and a kick button, reproducing Figures R2 and R4.
- Console (done in this release): the seam invariance test (01·F) now returns three verdicts, pure seam, changed dynamics, or nearing a transition, using the dominant-period ratio and the sharpness Q , with a noise-driven rhythm, a closeness control and an autocorrelation panel in units of each side’s own dominant period.
- Unfolding Bench (done in this release): an early-warning panel (03·B) applying the surrogate-tested sliding-window procedure of R6 to a drifting rhythm, a steady control, or any signal from instrument 01, with a visual-coding strip that draws each window as a closed loop shaded by its rank on the change indicators; and in the inverse Loewner bench (05), a crossover-scale readout with a $\times 2$ stretch test.
- Problem set (done): (7) read DET, LAM and $L_{\max}$ for the three presets and say which measure identifies the animation; (8) break the splay lock under frequency spread and compare the threshold with Figure R4. Proposed further: (a) choose embedding dimension and delay for a given series by false nearest neighbors and mutual information, and explain what the recurrence plot of a keyed animation must look like; (b) derive the stability condition of Equation R1 and the relaxation time, and say which of the two the collapse frames can measure.

Acknowledgment. I thank Dr. Norbert Schwarzer for the suggestion that led to R6: to look for the existence of dominating scales, with the pointer to Domb and Green [R8, R9]. He has not reviewed this note or the instruments built on his suggestion, and any error in carrying it out is mine.

Notes and sources

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Bryan M. Ingram works at the intersection of aneutronic energy, water technology, and frontier measurement. Correspondence: bryaningram.com. This note is a method transfer written for critique; it is an exercise, not a finding. Dr. Jorgensen reviewed and approved it and the interactive pieces; Dr. Schwarzer has not reviewed it, and any error in applying the methods to a seam is mine. The views in it do not represent any laboratory, agency, or venture named on that site.

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